## UNIVERSITY OF TORONTO Faculty of Arts and Science AUGUST 2013 SPECIAL DEFFERED EXAMINATION ECO381H1S – Personnel Economics

**Duration: 2 hours** (Instructor: J. Kantarevic)

## Instructions

The test consists of seven questions, each worth five points. Please show all your work. Non-programmable calculators are allowed. Good luck!

- 1. The Ministry of Health wishes to design a contract to improve the wait time to see a primary care physician. The contract, which will be offered to each individual physician, may be based on the number of patients who could see the physician on the same day when they are sick. Explain how each of the following factors may affect the extent to which the physician pay should be tied to this performance measure when the physician action cannot be observed or verified:
  - (a) (1 point) Physicians rarely change their careers.
  - (b) (1 point) The number of sick patients is highly predictable.
  - (c) (1 point) The Ministry also cares about the quality of care.
  - (d) (1 point) Physicians perceive themselves as healers rather than contractors.
  - (e) (1 point) Physicians face significant costs of moving to another country.
- (a) Possibility to interact with the ministry over longer horizon; if the ministry can observe physicians' actions, it may be able to design a self-enforcing contract based on this action (i.e. based on a subjective evaluation measure). This contract may be more efficient than a contract based on an objective measure (e.g. wait time) it the objective measure can be gamed.
- (b) This will in general increase the extent to which the pay is tied to performance when physicians are at least partially risk averse because it reduces uncertainty in outcome.
- (c) This introduces a multitasking problem. If the physician is risk averse and both access and quality are important, P4P will introduce additional risk. To offset the increase in the risk premium that the ministry has to pay the physician, it will in general pay to reduce the extent to which pay is tied to performance.
- (d) This may indicate that intrinsic motivation is important. If there is a danger of crowding out this motivation by tying pay to performance, it may be important to reduce the power of financial incentives.
- (e) This would determine the physician outside option and will be reflected in the fixed component of pay to meet the participation constraint, but will not in general impact the extent to which pay is tied to performance.

- 2. (5 points) Consider a principal-agent relationship with an altruistic agent. Specifically, suppose that for each additional unit of output, the agent's utility increases by  $\alpha$ =0.25. The output is stochastically related to the agent's action according to q=e+u, where e is the agent's action that the principal cannot observe and u is a random variable with a mean of zero and a variance of 2. The agent's cost of action is c(e)=0.5 $e^2$ . The outside option is 0 for both the agent and the principal. The agent is risk averse with the coefficient of absolute risk aversion equal to 2, while the principal is risk neutral. The principal designs the contract of the form w=a+bq. Find the optimal values of a and b.
- The agent's certainty equivalent is  $E[U]=a+(\alpha+b)E[q]-0.5rVar[(\alpha+b)q]-c(e)$ , which is equal to  $a+(0.25+b)e-0.5(2)(0.25+b^2)(2)-0.5e^2$ .
- The incentive compatibility constraint is  $\partial E[U]/\partial e=0=0.25+b-e$ .
- The participation constraint is  $E[U]=0=a+(0.25+b)e-2(0.25+b)^2-0.5e^2$ , from which it follows that  $a=0.5e^2-(0.25+b)e+2(0.25+b)^2$ .
- The principal's expected payoff is E[V]=E[q]-E[w]=e-a-be.
- Substitute in E[V] for the incentive compatibility and participation constraints to get  $E[V] = 0.25 + b 0.5(0.25 + b)^2 2(0.25 + b)^2 + 0.25(0.25 + b)$ .
- The first-order condition for b yields 1-(0.25+b)-4(0.25+b)+0.25, which gives  $b^*=0$ .
- Further,  $e^* = \alpha + b = 0.25$ .
- In addition,  $a=0.5e^2-(0.25+b)e+2(0.25+b)^2=0.09375$ .
- Lastly, we get that E[w]=0.09375 and E[V]=e-a=0.25-0.09375>0.
- 3. (5 points) Stacey Mowbray, the Second Cup CEO, wishes to design a compensation contract for two of her vice presidents (VP). The output is given by e<sub>1</sub>+0.5u for the first VP and by e<sub>2</sub>-0.5u for the second VP, where e is effort that cannot be observed by Stacey and u is a random variable that is distributed uniformly on the [-2,2] interval. The cost of effort is 0.5e<sup>2</sup> for both VPs. The two VPs have the same outside option of 1.5, while Stacey has the outside option of 0. Both VPs and Stacey are risk neutral. The VP with the higher output wins a promotion and an increase in salary from w to W, while the VP with the smaller output remains in his position with the same salary w. If the current position pays w=1, find the required salary in the new position that will induce VP<sub>1</sub> to provide the efficient level of effort.
  - Probability that VP 1 wins is  $p=prob(q_1>q_2)=prob(e_1+0.5u>e_2-0.5u)=pr(u>e_2-e_1)=F(e_2-e_1)=(2-e_2+e_1)0.25=0.5+0.25*(e_1-e_2).$
  - The expected utility of VP 1 is  $E[U]=pW+(1-p)w-0.5e_1^2=w+0.5[W-w]+0.25(e_1-e_2)[W-w]-0.5e_1^2$ . The first-order condition for  $e_1$  is then  $0.25[W-w]-e_1=0$ .
  - The efficient level of effort is determined by maximizing e₁-0.5e₁², which yields e₁\*=1.
  - Optimal W is then determined as follows. Given that w=1 and 0.25[W-w]- $e_1$ =0 and  $e_1$ \*=1, we have that 0.25[W-1]=1, or W\*=(1/.25)+1=5.

4. In 2011, parking enforcement officers in both Toronto and Montreal were compensated using a salary contract. In 2012, the compensation method for officers in Toronto changed to a pay-for-performance contract whereby officers' pay is directly related to the number of parking tickets issued. In Montreal, the officers' compensation method remained the salary contract. To evaluate the impact of changing the compensation method in Toronto, the City of Toronto collected data on the number of parking tickets issued in both cities in 2011 and 2012. The estimates from the regression model were as follows:

$$E[Y] = 100$$
 - 25Toronto +40Post +20Toronto×Post (1.4) (-2.1) (0.8) (2.5)

where Y indicates the number of parking tickets per 1,000 inhabitants, Toronto=1 for Toronto and 0 for Montreal, and Post=1 for year 2012 and 0 for year 2011, and the number in parentheses indicates the t-statistics.

- a. (1 point) Was there a significant difference in the average number of parking tickets between Toronto and Montreal in 2011?
- b. (1 point) Did Montreal experience a significant change in the average number of parking tickets between the two years?
- c. (2 points) What is the difference in the number of parking tickets in Toronto between 2012 and 2011? How much of this difference can be attributed to the change in the compensation method for the parking enforcement officers?
- d. (1 point) What is the assumption required to interpret this regression model as a causal impact of the compensation method on the behaviour of parking enforcement officers in Toronto?
- (a) E[Y|T=1,Post=0]-E[Y|T=0, Post=0]=(100-25)-100=-25. This is statistically significant given the t-statistic of -2.1.
- (b) E[Y|T=0,Post=1]-E[T=0,Post=0]=(100+40)-100=40. This is not statistically significant since t=0.8.
- (c) E[Y|T=1,Post=1]-E[Y|T=1,Post=0]=(100-25+40+20)-(100-25)=60. The difference in difference estimate is E[Y|T=1,Post=1]-E[Y|T=1,Post=0] -{E[Y|T=0,Post=1]-E[T=0,Post=0]} = 60-40=20. Therefore, 20 of change can be attributed to the change in the compensation method.
- (d) The common trend assumption: the difference in the outcome between Toronto and Montreal would remain constant over time if the compensation method in Toronto did not change.

- 5. Discuss the following statements:
  - (a) (1 point) It is always better to fill a new position by hiring from outside the firm rather than by promoting within the firm.
  - (b) (1 point) Some employment relationships are inefficient, even when the employer can perfectly observe and verify the actions of her employees.
  - (c) (1 point) New firms with uncertain prospects may have more difficulty motivating their employees than well-established firms.
  - (d) (1 point) Moral hazard arises whenever the principal delegates a task to the agent.
  - (e) (1 point) It is important that the legal system functions properly whenever employees' actions cannot be verified.
- (a) Promotion within (i.e. tournament) may reduce co-operation and encourage sabotage between employees. If this is a significant concern, hiring from outside is better.
- (b) It may be possible if outside options of both P and A are sufficiently high.
- (c) This may affect firm's reputation and impact their ability to offer self-enforcing contracts based on non-verifiable performance measures. This could be problematic if the available objective performance measures are highly dysfunctional.
- (d) No, only if P cannot observe and verify A's action and P and A have conflicting goals.
- (e) No, since contracts based on non-verifiable measures must be self-enforcing and cannot be enforced in courts even if the legal system functions perfectly.

- 6. (5 points) The basketball player's contribution to the team is given by q=e<sub>1</sub>+e<sub>2</sub>, where e<sub>1</sub> represents effort expended on scoring goals and e<sub>2</sub> represents effort expended on assisting others to score goals. The team manager can't observe either e<sub>1</sub> or e<sub>2</sub>. However, he can observe the number of goals scored (y<sub>1</sub>) and the number of assists (y<sub>2</sub>). The number of goals scored is given by y<sub>1</sub>=e<sub>1</sub>+u<sub>1</sub>, where u<sub>1</sub> is a random variable with a mean of zero and a variance of 1. Similarly, the number of assists is given by y<sub>2</sub>=e<sub>2</sub>+u<sub>2</sub>, where u<sub>2</sub> is a random variable with a mean of zero and a variance of 1. Assume that u<sub>1</sub> and u<sub>2</sub> are not correlated. The player is risk averse with a coefficient of absolute risk aversion of 2 and a cost of effort function c(e<sub>1</sub>,e<sub>2</sub>)=0.5(e<sub>1</sub>+e<sub>2</sub>)<sup>2</sup>. The contract specifies that the player's total compensation equals w=a+b<sub>1</sub>y<sub>1</sub>+b<sub>2</sub>y<sub>2</sub>. Find the player's risk premium if the manager designs an optimal contract.
- The risk premium is given by RP=0.5rVar[w]=0.5(2)( $b_1^2+b_2^2$ )=  $b_1^2+b_2^2$ .
- The expected utility:  $E[U]=a+b_1e_1+b_2e_2-b_1^2-b_2^2-0.5(e_1+e_2)^2$ .
- The first-order conditions are then (for  $e_1$ )  $b_1=e_1+e_2$  and (for  $e_2$ )  $b_2=e_1+e_2$ . From this, it follows that  $b_1=b_2$ . Let us define b as this common value, and e as the sum of  $e_1$  and  $e_2$ . We can then write the first-order conditions as e=b and the expected utility as  $E[U]=a+be-2b^2-0.5e^2=a-1.5b^2$ .
- The participation constraint can be written as E[U]=0, or a=1.5b<sup>2</sup>.
- The principal's expected payoff is equal to E[V]=e<sub>1</sub>+e<sub>2</sub>-a-b<sub>1</sub>e<sub>1</sub>-b<sub>2</sub>e<sub>2</sub>=e-a-be=b-1.5b<sup>2</sup>-b<sup>2</sup>=b-2.5b<sup>2</sup>.
- The first-order condition for b is then 1-5b=0, from which it follows that b=1/5.
- The optimal risk premium is then RP=  $b_1^2+b_2^2=2b^2=2(1/5)^2=2/25=0.08$ .

- 7. (5 points) Two lawyers are considering a potential partnership. Each lawyer is currently self-employed and generates revenues q according to q=e+0.5u, where e is effort and u is a random variable with a mean of zero. Both lawyers have the same cost of effort function  $c(e)=0.5\theta e^2$ , where  $\theta=1$  if the lawyer is self-employed and  $\theta<1$  if the lawyer is in the partnership. Both lawyers are risk neutral and have an outside option of zero. If the lawyers decide to form the partnership, the total revenues  $Q=q_1+q_2$  will be split equally between the partners. Show that both lawyers would be willing to form the partnership if and only if  $\theta \leq 0.75$ .
- In self-employment, each lawyer maximizes E[U]=E[q]-c(e)=e-0.5e<sup>2</sup>. The first-order condition for e is then 1-e=0, from which it follows that e\*=1 and E[U(e\*)]=1-0.5(1)<sup>2</sup>=0.5.
- In the partnership, each lawyer chooses his effort to maximizes  $0.5E[Q]-c(e)=0.5(e_1+e_2)-0.5\theta e_1^2$ . The first-order condition is then  $0.5-\theta e_1=0$ , from which it follows that  $e_1^*=1/2\theta$ .
- The expected utility is then  $E[U]=0.5[1/2\theta+1/2\theta]-0.5\theta(1/2\theta)^2=3/8\theta$ .
- Therefore, the partnership is preferred by each lawyer if and only if  $3/8\theta > 1/2$ , or  $\theta < 3/4$ .

THE END